DYNAMIC EQUATION OF STATE OF A GAS MIXTURE CONTAINING SOLID PARTICLES

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A dynamic equation of state is derived, describing the relationship between the pressure and volume of a thermal-nonequilibrium gas mixture. The methods of nonequilibrium thermodynamics are used to calculate the relaxation time.

Processes of adiabatic compression and expansion of a gas containing solid and liquid particles are of definite practical importance [1-4]. The pressure—volume relationship in these processes is approximated by a polytropic dependence with a certain effective polytropy exponent; both equilibrium systems and media with temperature and velocity retardation of the particles are considered in this connection. A number of special results on the polytropy exponents of disperse systems have been generalized in [4]. For the analysis of the processes and certain thermodynamic properties of a gas mixture with solid particles Yasnikov [5] and Tolmachev [6] have used the relaxation formalism of the thermodynamics of irreversible processes [7-10].

The objective of the present study is to investigate the behavior of a mixture of a gas with solid particles on the basis of a dynamic equation of state. In deriving this equation we consider the gas to be ideal and assume that heat transfer with the solid particles takes place according to Newton's law. Introducing the deviations of the temperatures T and T_s of the gas and particles from the equilibrium value T_0, $\xi_1 = T - T_0$, $\xi_2 = T_s - T_0$, we write the system entropy equation

$$S = C_x \ln (T_0 + \xi_1) + C_s \ln (T_0 + \xi_2) + f(x) + \text{const.} \tag{1}$$

Here x = p for an isobaric system, and x = V for an isochoric system. Following de Groot and Mazur [8], we write the entropy derivative in the form

$$\frac{dS}{dt} = -\frac{\mathbf{A}}{T} \cdot \frac{d\xi}{dt} \tag{2}$$

The affinity of each of the relaxation processes represents an element of the matrix $\mathbf{A} = \mathbf{T} \cdot \mathbf{g} \cdot \boldsymbol{\xi}$, where

$$\mathbf{g} = -\left(\frac{\partial^2 S}{\partial \xi \, \partial \xi} \right)_{\xi=0} = \begin{pmatrix} C_x T_0^{-2} & 0 \\ 0 & C_s T_0^{-2} \end{pmatrix}; \quad \xi = \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix}. \tag{3}$$

Expression (1) subject to the condition x = const is used in calculating the matrix elements g_{ij}. We write the phenomenological relations for the relaxation parameters $\boldsymbol{\xi}$ in the form

$$\frac{d\xi}{dt} = -\mathbf{L} \cdot \frac{\mathbf{A}}{T} = -\mathbf{M} \cdot \xi, \tag{4}$$

where the matrix **M** is related to the matrix of kinetic coefficients **L** by the equation $\mathbf{M} = \mathbf{L} \cdot \mathbf{g}$. The kinetic coefficients can be evaluated on the basis of experimental data, but it is more practical to determine the heat-transfer coefficients α between the solid and gaseous phases. In light of this fact and relation (3) we write

$$\mathbf{L} = \begin{Vmatrix} \dfrac{\alpha_0 T_0^2}{C_x^2} & -\dfrac{\alpha_0 T_0^2}{C_x C_s} \\ -\dfrac{\alpha_0 T_0^2}{C_x C_s} & \dfrac{\alpha_0 T_0^2}{C_s^2} \end{Vmatrix}, \quad \mathbf{M} = \begin{pmatrix} \alpha_0 C_x^{-1} & -\alpha_0 C_x^{-1} \\ -\alpha_0 C_s^{-1} & \alpha_0 C_s^{-1} \end{pmatrix}, \tag{5}$$

where $\alpha_0 = \alpha f^{(s)}$, α is the heat-transfer coefficient, and $f^{(s)}$ is the area of the heat-transfer surface. The eigenvalues of the matrix **M** determine the spectrum of relaxation times. Inasmuch as detM = 0, the characteristic polynomial of **M** has the form

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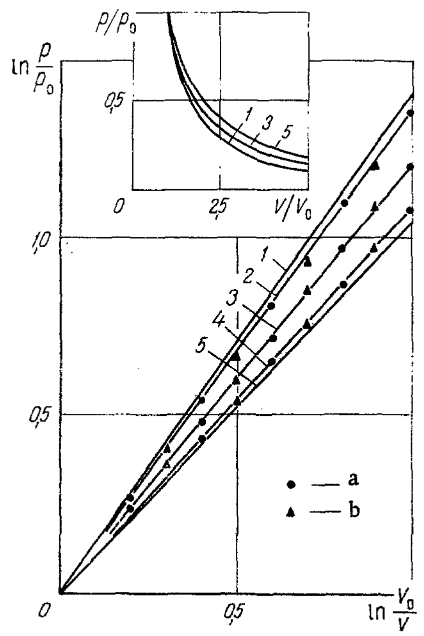

Fig. 1. Adiabatic compression (expansion) of a gas—solid particles mixture at a particle concentration $\mu = 5$. a) Data of [5]; b) data of [12]; 1) $k = 1.4$ (pure gas); 2) $k_T = 1.370$ ($\tau_V/\tau_0 = 10$); 3) 1.206 ($\tau_V/\tau_0 = 1$); 4) 1.069 ($\tau_V/\tau_0 = 0.1$); 5) $k_0 = 1.050$ (equilibrium compression).

$$\lambda^2 - \lambda \mathrm{Sp}\,\mathbf{M} = \lambda^2 - \lambda\alpha f^{(s)}\left(\frac{1}{C_x} + \frac{1}{C_s}\right). \tag{6}$$

The roots of this polynomial determine the isochoric-adiabatic and isobaric-adiabatic ($x = V$ and $x = p$, respectively) relaxation times:

$$\tau_x = \frac{1}{\lambda};\quad \tau_V = \left[\alpha f^{(s)}\left(\frac{1}{C_V} + \frac{1}{C_s}\right)\right]^{-1};\qquad \tau_p = \left[\alpha f^{(s)}\left(\frac{1}{C_p} + \frac{1}{C_s}\right)\right]^{-1} \tag{7}$$

In the analysis of isochoric-adiabatic thermal relaxation of a polydisperse medium in [6] the type of affinity of the process is postulated, and the relaxation parameters are selected accordingly; in principle, this approach is not required in conjunction with the formalism discussed above. Practical application of the relaxation spectrum of a polydisperse mixture is rendered exceedingly difficult by the need to determine the heat-transfer coefficient between the gas and each fraction of solid material, particularly in concentrated systems. Quite possibly methods similar to those developed in [11] could prove useful here.

If $\xi_1 = 0$, then expression (4) acquires the following form with regard for (3) and (5):

$$C_s \frac{dT_s}{dt} = \alpha f^{(s)}(T - T_s). \tag{8}$$

For $\xi_2 = 0$, rather than (7), we obtain the isochoric-isothermal relaxation time $\tau_{TV} = C_V \cdot (\alpha f^{(s)})^{-1}$ and the isobaric-isothermal relaxation time $\tau_{Tp} = C_p(\alpha f^{(s)})^{-1}$ of the gaseous phase.

Writing the first law of thermodynamics for the given mixture, along with the equation of state of the gas,

$$C_s \frac{dT_s}{dt} = C_p \frac{dT}{dt} - V \frac{dp}{dt}, \tag{9}$$

$$pV = R_0 T,\ R_0 = MR, \tag{10}$$

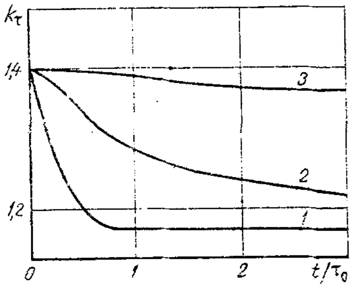

Fig. 2

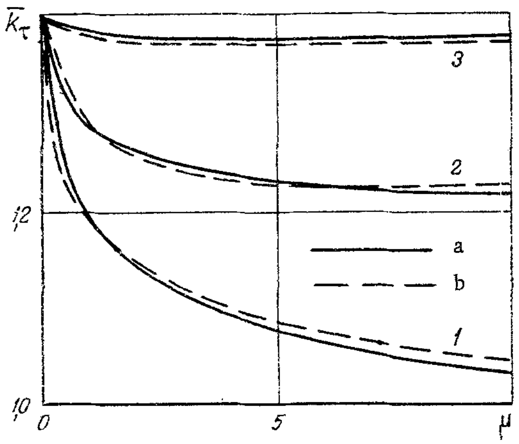

Fig. 3

Fig. 2. Effective adiabatic exponent versus time for $\mu = 1$. 1) $\tau_V/\tau_0 = 0.1$; 2) 1; 3) 10.

Fig. 3. Average adiabatic exponent $\bar{k}_\tau$ versus particle concentration. a) Results of this study; b) [5]; curves 1-3, same as in Fig. 2.

together with (8) we obtain a closed system of equations describing the relationship between the volume and pressure in compression or expansion of the gas suspension. Eliminating the gas and particle temperatures T and T_s from these equations, after straightforward but rather cumbersome transformations we obtain the dynamic equation of the process:

$$\frac{d^2p}{dt^2}V + \frac{dp}{dt}\left[(k+1)\frac{dV}{dt} + \frac{V}{\tau_V}\right] + pk\left(\frac{d^2V}{dt^2} + \frac{1}{\tau_p}\cdot\frac{dV}{dt}\right) = 0. \tag{11}$$

In this equation V(t) is a given function of the time, τ_V and τ_p are described by relations (7), and $k = C_p/C_V$ is the adiabatic exponent of the pure gas.

For large relaxation times ($\tau_x \to \infty$) Eq. (11) is satisfied by the pure-gas adiabat pV^k = const, and for small relaxation times by the adiabat for the equilibrium mixture pV^{k_0} = const, where

$$k_0 = \frac{k + C_sC_V^{-1}}{1 + C_sC_V^{-1}} = k\frac{\tau_V}{\tau_p}. \tag{12}$$

If the volume variation is specified by a function of the form

$$V = V_0\exp[-\beta t], \tag{13}$$

then Eq. (11) takes the form

$$\frac{d^2p}{dt^2} - \frac{dp}{dt}\left[(k+1)\beta - \frac{1}{\tau_V}\right] + p\left[k\beta^2 - \frac{k_0\beta}{\tau_V}\right] = 0, \tag{14}$$

where k_0 is given by relation (12).

Assuming that only the pure gas is compressed at the initial time, we obtain the initial condition for the derivative

$$\left(\frac{dp}{dt}\right)_{t=0} = k\beta p_0, \tag{15}$$

where $p_0 = p(t = 0)$. Now the solution of Eq. (14) is written as follows:

$$p(t) = p_0\left(\frac{\lambda_1 - \beta k}{\lambda_1 - \lambda_2}\exp[\lambda_2 t] - \frac{\lambda_2 - \beta k}{\lambda_1 - \lambda_2}\exp[\lambda_1 t]\right), \tag{16}$$

where λ_1 and λ_2 are the roots of the characteristic equation

$$\lambda_{1,2} = \frac{1}{2}(k+1)\beta - \frac{1}{2\tau_V} \pm \sqrt{\left[\frac{1}{2}(k+1)\beta - \frac{1}{2\tau_V}\right]^2 - k\beta^2 + \frac{k_0\beta}{\tau_V}}. \tag{17}$$

Using expression (13), we eliminate the time from (16) and obtain a relation between the variations of the pressure and volume of the gas:

$$\frac{p}{p_0}=\frac{\lambda_1-\beta k}{\lambda_1-\lambda_2}\left(\frac{V_0}{V}\right)^{\frac{\lambda_2}{\beta}}-\frac{\lambda_2-\beta k}{\lambda_1-\lambda_2}\left(\frac{V_0}{V}\right)^{\frac{\lambda_1}{\beta}}. \tag{18}$$

This relation is plotted in Fig. 1 in logarithmic coordinates for a particle concentration $\mu = M_s/M = 5$ and a relative relaxation time $\tau_V/\tau_0 = 0.1$ to 10. The characteristic time τ_0 of the process is simply related to the quantity β entering into the expression describing exponential time variation of the volume: $\tau_0 = 1/\beta$. It is evident from the figure that as τ_V/τ_0 is increased from 0 to ∞ the polytropy exponent increases from k_0 (for equilibrium compression) to $k = 1.4$ (for compression of the pure gas). Calculations show that the disparity between the results of [5, 12], represented by points in Fig. 1, and our own data does not exceed 2%.

Figure 2 gives the time dependence of the polytropy exponent for various ratios τ_V/τ_0 at $\mu = 1$, and Fig. 3 gives the dependence of k_τ on μ for various τ_V/τ_0. Here again we observe good agreement with the data of [5], represented by dashed curves in the figure.

Expression (17) can be written in the form

$$\lambda_{1,2}=\frac{1}{2}\beta\left\{k+1-\frac{\tau_0}{\tau_V}\pm\sqrt{\left[(k+1)-\frac{\tau_0}{\tau_V}\right]^2-k+k_0\frac{\tau_0}{\tau_V}}\right\}, \tag{19}$$

whence it follows that for $\tau_V/\tau_0 > 10$ terms containing the relative isochoric relaxation time can be neglected. Then $\lambda_1 = \beta k$, $\lambda_2 = \beta$, $k_\tau = k$; i.e., the mixture is compressed in the same way as the pure gas.

An analysis of the results of pressure calculations according to expressions (16) and (17) shows that the compression can be regarded as equilibrium for $\tau_V/t < 10$, where t is the duration of the process, and for a relative relaxation time $\tau_V/\tau_0 < 1$. Here the average polytropy exponent of the relaxation process differs from the equilibrium value k_0 by not more than 5%.

In all other cases, i.e., for $1 < \tau_V/\tau_0 < 10$ and $\tau_V/t > 10$, the exact solution (16), (17) must be used to calculate the parameters of the relaxation process.

NOTATION

T, T_s, temperatures of gas and particles, respectively; T_0, equilibrium temperature; p, V, pressure and volume of gas; R, gas constant; S, entropy of the system; C_s, total heat capacity of the particle material; C_p, C_V, total heat capacities of the gas at p = const and V = const; $\mu = M_s/M$, fraction of particles associated with unit mass of the gas; M, mass of gas; M_s, mass of particles; M, matrix of affinities of relaxation processes; L, matrix of kinetic coefficients; α, heat-transfer coefficient; $f^{(s)}$, area of heat-transfer surface; ξ_i, deviation of i-th parameter from its equilibrium value; t, time; τ_V, τ_p, isochoric-adiabatic and isobaric-adiabatic relaxation times; τ_{TV}, τ_{Tp}, isochoric-isothermal and isobaric-isothermal relaxation times; k, adiabatic exponent of pure gas; k_0, adiabatic exponent for equilibrium compression of gas–solid particles mixture; k_τ, effective adiabatic exponent (polytropy exponent) for nonequilibrium compression (expansion) of mixture.

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IMPACT OF PARTICLES OF A GRANULAR MATERIAL ON A HARD SURFACE

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The recovery coefficient for the particle velocities of a number of granular materials of different sizes and shapes are determined experimentally.

In mathematical simulation of the motion of pulverized materials, it is sometimes necessary to calculate the motion of particles after their impact on the apparatus walls. In this, it is necessary to know the recovery coefficient of the normal and the tangential components of the particle velocity $k = w_{2n}/w_{1n}$ and $k_t = w_{2t}/w_{1t}$ and often their derivatives: the recovery coefficient of the total velocity $n = w_2/w_1$ and the ratio of angles $\varepsilon = \alpha_2/\alpha_1$. The problem of imperfectly elastic collision has been solved theoretically [1-3] only for spheres with a sufficiently large diameter, where the surface roughness can be neglected, while the recovery coefficient, which depends on the elastic characteristics of the particle and the surface, is either assigned or determined for an absolutely smooth surface. For the most important case — fine particles of irregular shape — a theoretical solution is apparently impossible, and the problem consists in obtaining reliable experimental relationships for certain materials with particles of any specific shape or slightly varying shapes. We shall present here the results of an experimental investigation of the rebound of particles with spherical and irregular shapes in relation to the particle dimension δ, the impact velocity w_1, and the impact angle α_1.

We used small glass balls with the density $\rho = 6600$ kg/m^3 and small polystyrene balls ($\rho = 1080$ kg/m^3), sorted by means of screens into the narrow particle size ranges δ = 40-45; 45-50; 50-56; 56-63; 63-70; 70-80; 80-90; 90-100; 100-125; 125-160 μm (glass) and 315-400; 400-500; 500-630 μm (polystyrene), and also calibrated steel balls for ball bearings with diameters of 680 and 1000 μm ($\rho = 7960$ kg/m^3). For particles of irregular shape, we used coal (anthracite) particles in the same size ranges with $\rho = 1350$ kg/m^3 (within 160-630 μm), corundum particles with $\rho = 3900$ kg/m^3 (100-315 μm), and ferrochrome particles with $\rho = 6910$ kg/m^3 (80-200 μm). The uniformity of the shapes of coal, corundum, and ferrochrome particles has been confirmed by means of reflected-light microscopy. The ratio of the maximum grain size to the minimum size is equal to 2-3. This can probably be explained by the homogeneity of these materials and the fact that the same method of crushing (in a ball mill) is used. The diameter of a sphere with the equivalent mass, determined by weighing 300-1000 particles, is used as the characteristic size of a particle size range. The characteristic size is equal to the arithmetic mean of sizes at the extremes of a range with a scatter of ±6%.

The particles are dropped vertically downward on an inclined ground steel plate at a velocity $w_1 = 0.5$–30 m/sec at an angle $\alpha_1 = 15$–$70°$ with respect to the normal to the plate at the point of impact. Their trajectories in the form of successive points are recorded on aerial photographic film, using illumination from an IFP-5000 flash bulb, supplied from a special source [4]. The flashing frequency is set within the range 200-500 Hz, depending on the impact velocity. The velocities w_1 and w_2 are determined with respect to the spacing between the points with an error of 2.5%, while the angles α_1 and α_2 are determined with respect to tangents to the trajectories at the impact point with an error of 3%. The values of k, k_t, n, and ε for each range of particle sizes of the material under investigation are obtained by statistically processing 20-30 trajectories for spheres and 30-50 trajectories for particles of irregular shape.

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